

Scalar-tensor cosmological simulations

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We present N -body cosmological simulations in the framework of the Newtonian limit of scalar-tensor theories of gravity. The scalar field is described by a modified Helmholtz equation with a source that is coupled to the standard Poisson equation of Newtonian gravity. The effective gravitational force is given by two contributions: the standard Newtonian potential plus a Yukawa potential stemming from massive scalar fields. In particular, we consider simulations of Λ CDM models and compute the density and velocity profiles of the most massive groups found at $z=0$.

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I. INTRODUCTION

In this work we present some preliminary results about the role scalar fields play in cosmological simulations, in particular on the process of large scale structure formation. Scalar fields have been around for so many years since the pioneering work of Jordan, Brans, and Dicke[1, 2]. Nowadays they are considered as a mechanism for inflation[3]; the dark matter component of galaxies[4]; the quintessence field to explain dark energy in the universe[5]. The main goal of this work is to study the large scale structure formation where the usual approach is that the evolution of the initial primordial fluctuation energy density fields evolve following Newtonian mechanics in an expanding background[6]. The force between particles are the standard Newtonian gravitational force. Now, we will see that we can introduce the scalar fields by adding a term in this force. This force will turn out to be of Yukawa type with two parameters (α, λ) [7]. For so many years this kind of force, the so called fifth force, was thoroughly studied theoretically[8] and many experiments were done to constrain the Yukawa parameters[9]. We have also been studying, in the past years, the effects of this kind of force on some astrophysical phenomena[7, 10, 11, 12]. The Yukawa force comes as a Newtonian limit of a scalar-tensor theory with the scalar field non-minimally coupled to gravitation[13] although other alternatives can be found[14]. It is our purpose to find the role these scalar fields play on the large scale structure formation processes. We start by discussing the standard LCDM model and the general approach in N -body simulations (See Bertschinger[15] for details). Then, we present the modifications we need to do to consider the effects of a static scalar field and we show the results of this theory for the cosmological concordance model of a Λ CDM universe[16]. To perform the simulations we have modified a standard serial treecode the author has developed [17] and the Gadget 1 [18] (see also <http://www.astro.inin.mx/mar>) in order to take into account the contribution of the Yukawa potential.

II. EVOLUTION EQUATIONS FOR A Λ CMD UNIVERSE

A. Newtonian approximation

The study of large-scale formation in the universe is greatly simplified by the fact that a limiting approximation of general relativity, Newtonian mechanics, applies in a region small compared to the Hubble length cH_0^{-1} ($cH_0^{-1} \approx 3000h^{-1}$ Mpc, where c is the speed of light, $H_0 = 100h$ km/s/Mpc, is Hubble's constant and $h \approx (0.5 - 1)$), and large compared to the Schwarzschild radii of any collapsed objects. The rest of the universe affect the region only through a tidal field. The length scale cH_0^{-1} is of the order of the largest scales currently accessible in cosmological observations and $H_0^{-1} \approx 10^{10}h^{-1}$ yr characterizes the evolutionary time scale of the universe.

The Newtonian approximation can fail at much smaller R if the region includes a compact object like a neutron star or black hole, but one can deal with this by noting that at distances large compared to the Schwarzschild radius the object acts like an ordinary Newtonian point mass. It is speculated that in nuclei of galaxies there might be black holes as massive as $10^9 M_\odot$, Schwarzschild radius $\sim 10^{14}$ cm. If this is an upper limit, Newtonian mechanics is a good approximation over a substantial range of scales, 10^{14} cm $\ll r \ll 10^{28}$ cm.

B. General Scalar-tensor theory

Let us consider a typical scalar–tensor theory given by the following Lagrangian

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \left[-\phi R + \frac{\omega(\phi)}{\phi} (\partial\phi)^2 - V(\phi) \right] + \mathcal{L}_M(g_{\mu\nu}) , \quad (1)$$

Here $g_{\mu\nu}$ is the metric, $\mathcal{L}_M(g_{\mu\nu})$ is the matter Lagrangian and $\omega(\phi)$ and $V(\phi)$ are arbitrary functions of the scalar field. From Lagrangian (1) we get the gravitational equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{\phi} \left[8\pi T_{\mu\nu} + \frac{1}{2}Vg_{\mu\nu} + \frac{\omega}{\phi} \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\frac{\omega}{\phi}(\partial\phi)^2 g_{\mu\nu} + \phi_{;\mu\nu} - g_{\mu\nu}\square\phi \right] , \quad (2)$$

and the scalar field equation

$$\square\phi + \frac{\phi V' - 2V}{3 + 2\omega} = \frac{1}{3 + 2\omega} [8\pi T - \omega'(\partial\phi)^2] , \quad (3)$$

where $()' \equiv \frac{\partial}{\partial\phi}$. The gravitational constant is now contained in $V(\phi)$ and the scalar field get a mass m_{SF} .

C. Newtonian approximation of STT

In the present study, however, we want to consider the influence of scalar fields in the limit of a static STT, and therefore we need to describe the theory in its Newtonian approximation, that is, where gravity and the scalar fields are weak (and time independent) and velocities of stars are non-relativistic. We expect to have small deviations of the scalar field around the background field, defined here as $\langle\phi\rangle$ and can be understood as the scalar field beyond all matter. If one defines the perturbations $\phi = \langle\phi\rangle + \bar{\phi}$ and $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric, the Newtonian approximation gives [13]

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = \frac{G_N}{1 + \alpha} 4\pi\rho - \frac{1}{2}\nabla^2 \bar{\phi} , \quad (4)$$

$$\nabla^2 \bar{\phi} - m_{SF}^2 \bar{\phi} = -8\pi\alpha\rho , \quad (5)$$

we have set $\langle\phi\rangle = (1 + \alpha)/G_N$ and $\alpha \equiv 1/(3 + 2\omega)$. In the above expansion we have set the cosmological constant term equal to zero, since on galactic scales its influence should be negligible. We only consider the influence of dark matter due to the boson field of mass m_{SF} governed by Eq. (5), that is the modified Helmholtz equation. Equations (4) and (5) represent the Newtonian limit of STT with arbitrary potential $V(\phi)$ and function $\omega(\phi)$ that where Taylor expanded around $\langle\phi\rangle$. The resulting equations are then distinguished by the constants G_N , α , and $\lambda = h_P/m_{SF}c$. Here h_P is Planck's constant.

Note that Eq. (4) can be cast as a Poisson equation for $\psi \equiv (1/2)(h_{00} + \bar{\phi}/\langle\phi\rangle)$,

$$\nabla^2 \psi = 4\pi \frac{G_N}{1 + \alpha} \rho . \quad (6)$$

The next step is to find solutions for this new Newtonian potential given a density profile, that is, to find the so-called potential–density pairs. General solutions to Eqs. (5) and (6) can be found in terms of the corresponding Green functions, and the new Newtonian potential is

$$\Phi_N \equiv \frac{1}{2}h_{00} = -\frac{G_N}{1 + \alpha} \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)}{|\mathbf{r} - \mathbf{r}_s|} - \alpha \frac{G_N}{1 + \alpha} \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s) e^{-|\mathbf{r} - \mathbf{r}_s|/\lambda}}{|\mathbf{r} - \mathbf{r}_s|} + \text{B.C.} \quad (7)$$

The first term of Eq. (7), given by ψ , is the contribution of the usual Newtonian gravitation (without scalar fields), while information about the scalar field is contained in the second term, that is, arising from the influence function determined by the modified Helmholtz Green function, where the coupling ω (α) enters as part of a source factor.

D. Cosmological evolution equations using a static STT

To simulate cosmological systems, the expansion of the universe has to be taken into account. Also, to determine the nature of the cosmological model we need to determine the composition of the universe, i. e., we need to give the values of Ω_i for each component i , taking into in this way all forms of energy densities that exist at present.

If a particular kind of energy density is described by an equation of state of the form $p = w\rho$, where p is the pressure and w is a constant, then the equation for energy conservation in an expanding background, $d(\rho a^3) = -pd(a^3)$, can be integrated to give $\rho \propto a^{-3(1+w)}$. Then, the Friedmann equation for the expansion factor $a(t)$ is written as

$$\frac{\dot{a}^2}{a^2} = H_0^2 \sum_i \Omega_i \left(\frac{a_0}{a} \right)^{3(1+w_i)} - \frac{k}{a^2} \quad (8)$$

where w_i characterizes equation of state of specie i . The most familiar forms of energy densities are those due to pressureless matter with $w_i = 0$ (that is, nonrelativistic matter with rest-mass-energy density ρc^2 dominating over the kinetic-energy density $\rho v^2/2$) and radiation with $w_i = 1/3$. The density parameter contributed today by visible, nonrelativistic, baryonic matter in the universe is $\Omega_B \approx (0.01 - 0.2)$ and the density parameter that is due to radiation is $\Omega_R \approx 2 \times 10^{-5}$. In this work we will consider a model with only two energy density contribution. One which is a pressureless and nonbaryonic dark matter with $\Omega_{DM} \approx 0.3$ that does not couple with radiation. Other, that will be a cosmological constant contribution $\Omega_\Lambda \approx 0.7$ with and equation of state $p = -\rho$. The above equation for $a(t)$ becomes

$$\frac{\dot{a}^2}{a^2} = H_0^2 \left[\Omega_{DM} \left(\frac{a_0}{a} \right)^3 + \Omega_\Lambda \right] - \frac{k}{a^2} \quad (9)$$

Here, we employ a cosmological model with a static scalar field which is consistent with the Newtonian limit given by Eq. (7). Thus, the scale factor, $a(t)$, is given by the following Friedman model,

$$a^3 H^2 = H_0^2 \left[\frac{\Omega_{m0} + \Omega_{\Lambda0} a^3}{1 + \alpha} + \left(1 - \frac{\Omega_{m0} + \Omega_{\Lambda0}}{1 + \alpha} \right) a \right] \quad (10)$$

where $H = \dot{a}/a$, Ω_{m0} and $\Omega_{\Lambda0}$ are the matter and energy density evaluated at present, respectively. We notice that the source of the cosmic evolution is deviated by the term $1 + \alpha$ when compared to the standard Friedman-Lemaitre model. Therefore, it is convenient to define a new density parameter by $\Omega_i^\alpha \equiv \Omega_i/(1 + \alpha)$. This new density parameter is such that $\Omega_m^\alpha + \Omega_\Lambda^\alpha = 1$, which implies a flat universe, and this shall be assumed in our following computations, where we consider $(\Omega_m^\alpha, \Omega_\Lambda^\alpha) = (0.3, 0.7)$. For positive values of α , a flat cosmological model demands to have a factor $(1 + \alpha)$ more energetic content (Ω_m and Ω_Λ) than in standard cosmology. On the other hand, for negative values of α one needs a factor $(1 + \alpha)$ less Ω_m and Ω_Λ to have a flat universe. To be consistent with the CMB spectrum and structure formation numerical experiments, cosmological constraints must be applied on α in order for it to be within the range $(-1, 1)$ [19, 20, 21, 22].

In the Newtonian limit of STT of gravity, the Newtonian motion equation for a particle i is written as

$$\ddot{\mathbf{x}}_i + 2H \mathbf{x}_i = -\frac{1}{a^3} \frac{G_N}{1 + \alpha} \sum_{j \neq i} \frac{m_j (\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3} F_{SF}(|\mathbf{x}_i - \mathbf{x}_j|, \alpha, \lambda) \quad (11)$$

where \mathbf{x} is the comovil coordinate, and the sum includes all periodic images of particle j , and $F_{SF}(r, \alpha, \lambda)$ is

$$F_{SF}(r, \alpha, \lambda) = 1 + \alpha \left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \quad (12)$$

which, for small distances compared to λ , is $F_{SF}(r < \lambda, \alpha, \lambda) \approx 1 + \alpha \left(1 + \frac{r}{\lambda} \right)$ and, for long distances, is $F_{SF}(r > \lambda, \alpha, \lambda) \approx 1$, as in Newtonian physics.

We now analyze the general effect that the constant α has on the dynamics. The role of α in our approach is as follows. On one hand, to construct a flat model we have set the condition $\Omega_m^\alpha + \Omega_\Lambda^\alpha = 1$, which implies having $(1 + \alpha)$ times the energetic content of the standard Λ CDM model. This essentially means that we have an increment by a factor of $(1 + \alpha)$ times the amount of matter, for positive values of α , or a reduction of the same factor for negative values of α . Increasing or reducing this amount of matter affects the matter term on the r.h.s. of the equation of motion (11), but the amount affected cancels out with the term $(1 + \alpha)$ in the denominator of (11) stemming from the new Newtonian potential. On the other hand, the factor F_{SF} augments (diminishes) for positive (negative) values of α for small distances compared to λ , resulting in more (less) structure formation for positive (negative) values of α compared to the Λ CDM model. For $r \gg \lambda$ the dynamics is essentially Newtonian.

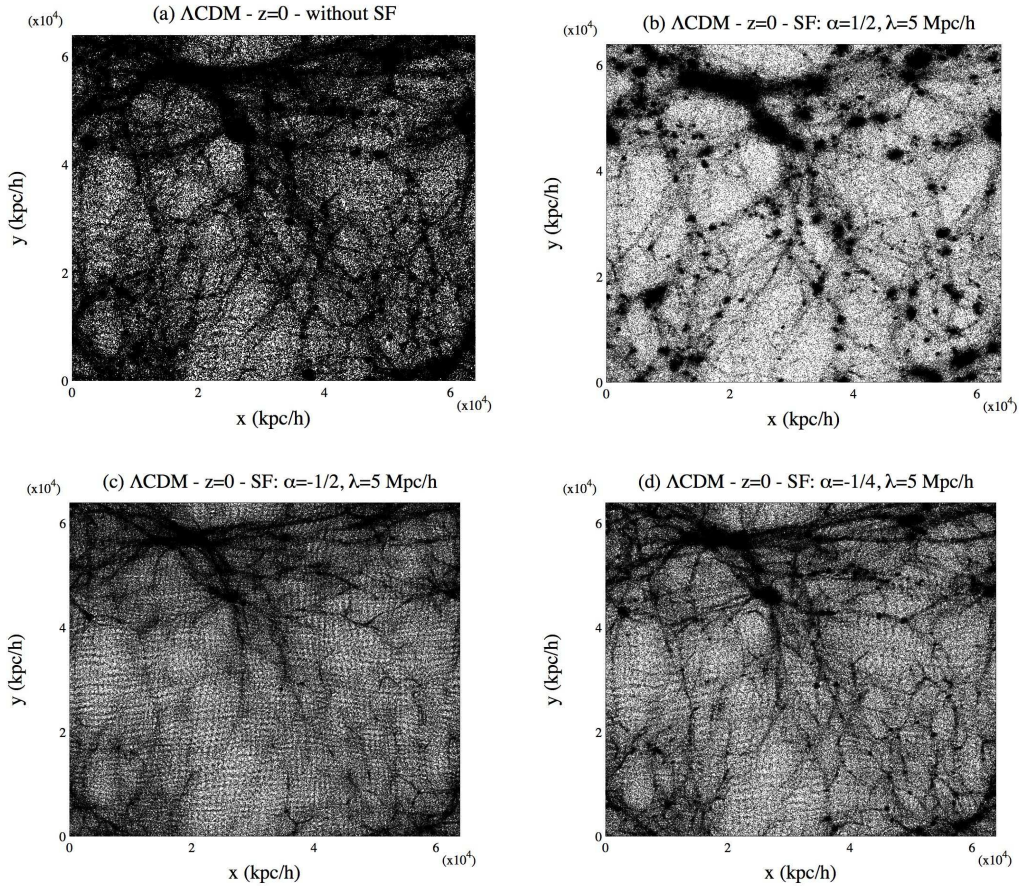


FIG. 1: x - y snapshots at $z = 0$ of a Λ CDM universe. See text for details.

III. RESULTS

In this section, we present results of the cosmological simulations of a Λ CDM universe with and without SF contribution. We consider the *smal box* initial condition in the Cosmic Data Bank web page (<http://t8web.lanl.gov/people/heitmann/test3.html>). The initial condition use a box 90 Mpc size and 256^3 particles. It is somewhat small as a representative simulation due to lack of large-scale power but straddle a representative range of force and mass resolutions for state-of-the-art large scale structure simulations designed to study power spectra, halo mass functions, weak lensing, and so on.

The initial linear power spectrum was generated using the fitting formula by Klypin & Holtzman[23] for the transfer function. This formula is a slight variation of the common BBKS fit[24]. It includes effects from baryon suppression but no baryonic oscillations. We use the standard Zel'dovich approximation[25] to provide the initial particle displacement off a uniform grid and to assign initial particle velocities. The starting redshift is $z_{in} = 50$ and we choose the following cosmology: $\Omega_m = 0.314$ (where Ω_{DM} includes cold dark matter and baryons), $\Omega_b = 0.044$, $\Omega_\Lambda = 0.686$, $H_0 = 71$ km/s/Mpc, $\sigma_8 = 0.84$, and $n = 0.99$. These values are in concordance with measurements of cosmological parameters by WMAP[26]. The simulations we present here use an initial condition with only 723,925 particles, obtained from the original initial condition by a reduction procedure based on a tree scheme. This implies that particle masses are in the order of $1.0 \times 10^{10} M_\odot$. The individual softening length was 20 kpc/h. These choices of softening length are consistent with the mass resolution set by the number of particles.

We now present results for the Λ CDM model previously described. Because the visible component is the smaller one and given our interest to test the consequences of including a SF contribution to the evolution equations, our model excludes gas particles, but all its mass has been added to the dark matter. We restrict the values of α to the interval $(-1, 1)$ [19, 20, 21, 22] and use $\lambda = 5$ Mpc/h, since this scale turns out to be an intermediate scale between the size of the clump groups and the separation of the formed groups.

In Fig. 1 we show x - y snapshots at redshift $z = 0$ of our Λ CDM model. Fig. 1 (a) presents the standard case

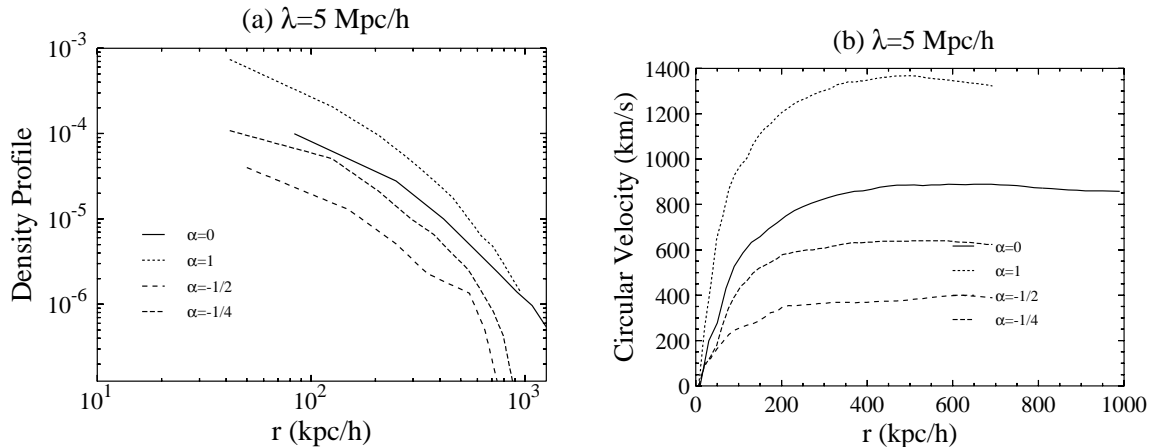


FIG. 2: (a) Density profiles for one of the most massive groups at $z = 0$ of a Λ CDM universe. The group is located approximately at $x = 14 \text{ Mpc}/h$, $y = 57.5 \text{ Mpc}/h$ in Fig 1(a). Vertical scale is in units of $\rho_0 = 10^{10} \text{ M}_\odot h^{-1} / (h^{-1} \text{ kpc})^3$. (b) The corresponding circular velocity.

without SF, i.e., the interaction between bodies is through the standard Newtonian potential. In (b) we show the case with $\alpha = 1/2$, $\lambda = 5 \text{ Mpc}/h$. In (c) $\alpha = -1/2$, $\lambda = 5 \text{ Mpc}/h$. In (d) $\alpha = -1/4$, $\lambda = 5 \text{ Mpc}/h$. One notes clearly how the SF modifies the matter structure of the system. The most dramatic cases are (b) and (c) where we have used $\alpha = 1/2$ and $\alpha = -1/2$, respectively. Given the argument at the end of last section, in the case of (b), for $r \ll \lambda$, the effective gravitational pull has been augmented by a factor of $3/2$, in contrast to case (c) where it has diminished by a factor of $1/2$; in model (d) the pull diminishes only by a factor of $3/4$. That is why one observes for $r < \lambda$ more structure formation in (b), less in (d), and lesser in model (c). The effect is then, for a growing positive α , to speed up the growth of perturbations, then of halos and then of clusters, whereas negative α values ($\alpha \rightarrow -1$) tend to slow down the growth.

Next, we found the groups in the system using a friend-of-friend algorithm and select one of the most massive ones. The chosen group is located approximately at $x = 14 \text{ Mpc}/h$, $y = 57.5 \text{ Mpc}/h$. The group was analyzed by obtaining their density profiles (Fig. 2(a)) and circular velocities (Fig. 2(b)). The more cuspy case is for $\alpha = 1/2$ and the less cuspy is for $\alpha = -1/2$. The circular velocity curves were computed using $v_c^2 = G_N M(r)/r$. The case with $\alpha = 1/2$ corresponds to higher values of v_c , since this depends on how much accumulated mass there is at a distance r and this is enhanced by the factor F_{SF} for positive values of α .

IV. CONCLUSIONS

In general, we can say that even though we have done first numerical simulations using non-minimally coupled SF, the analysis we have done is insufficient to give us a clear conclusions on the role played by SF in the large-scale structure formation process. We will need to do a systematic study of the evolution of the two-point correlation function which is a measure of galaxy clustering. We also will need to compute the mass power spectrum and velocity dispersions of the halos. Therefore, we will be able make sistematic comparisons with observations. However, and in favor of the model, the theoretical scheme we have used is compatible with local observations because we have defined the background field constant $\langle \phi \rangle = G_N^{-1}(1 + \alpha)$. A direct consequence of the approach is that the amount of matter (energy) has to be increased for positive values of α and diminished for negative values of α with respect to the standard Λ CDM model in order to have a flat cosmological model. Quantitatively, our model demands to have $\Omega/(1 + \alpha) = 1$ and this changes the amount of dark matter and energy of the model for a flat cosmological model, as assumed. The general gravitational effect is that the interaction including the SF changes by a factor $F_{SF}(r, \alpha, \lambda) \approx 1 + \alpha \left(1 + \frac{r}{\lambda}\right)$ for $r < \lambda$ in comparison with the Newtonian case. Thus, for $\alpha > 0$ the growth of structures speeds up in comparison with the Newtonian case. For the $\alpha < 0$ case the effect is to diminish the formation of structures. For $r > \lambda$ the dynamics is essentially Newtonian.

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